Specific-purpose globalizations for Newton's method: anisotropic optimization of curved meshes

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***Abstract*—We derive an optimization method to adapt straight-edged and curved piece-wise polynomial meshes to the stretching and alignment of a target metric. Two globalization strategies for the optimization method are proposed: backtracking line search and restricted trust region. To compare both globalization approaches, we derive a specific-purpose implementation of Newton's method for each globalization. To propose these two new implementations, we present different emulation methods to interchange between both approaches their non-shared globalization features. Once the number of non-linear iterations is comparable, we have been able to improve the inexact Newton implementation, with both globalization methods, to reduce one order of magnitude the total number of sparse matrix-vector products.**

I. Introduction

The optimization of an objective function that measures a global mesh quantity by changing the coordinates of the mesh nodes has been used in several meshing methods [1]. To optimize the objective function, we can iteratively modify the coordinates of either one free node (local) or all the free nodes (global) per non-linear iteration by using either gradient-based (first-order) or Hessian-based (second-order) optimization methods. Existent literature shares a common conclusion. That is, when highly optimized and accurate meshes are required, especially in isotropic meshes featuring high gradations of the element size, a specific-purpose global feasible Newton method outperforms one-node optimization methods.

Although the existent literature does not deal with piece-wise polynomial meshes and highly stretched elements, it provides valuable knowledge to devise our specific-purpose global optimization method. Our goal is to derive an optimization method to adapt straight-edged and curved piece-wise polynomial meshes to the stretching and alignment of a target metric. The objective function has been proposed before [2]. However, a specific purpose solver where an initial mesh configuration, with nodal coordinates not in the convergence basin of a local optimum, is efficiently driven to an optimal configuration has not been proposed. Accordingly, we are especially interested in a globalized solver that can deal with inaccurate initial approximations with low and high polynomial degrees and targets metrics featuring highly varying size gradations, stretching ratios, and principal directions. Thus, the main research question of this work arises whether it is better to use a backtracking line search or a restricted trust region.

To compare both globalization approaches for the adaption problem to a target metric problem, we derive two specific-purpose implementations of Newton's method equipped with backtracking line search (BLS) and restricted trust region (RTR) [3], [4].

II. Results

As a test example we consider the hexahedral domain $\Omega := [-\frac{1}{2}, \frac{1}{2}]^3$ equipped with the metric $\mathbf{M}$ given by

$$\mathbf{M} = \nabla\varphi^{\mathrm{T}} \cdot \mathbf{D} \cdot \nabla\varphi,$$

where

$$\varphi(x,y) := \left(x, \frac{10y - \cos(2\pi x)}{\sqrt{100+4\pi^2}}\right)$$

and where $\mathbf{D}$ is the three-dimensional boundary layer metric with its axes aligned according to the Cartesian axes with a stretching in the z-direction at the plane $z = 0$, that is,

$$\mathbf{D} := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/h(z)^2 \end{pmatrix}.$$

The function h is given by

$$h(x) := h_{\min} + \gamma|x|,$$

with $h_{\min} = 2 \cdot 10^{-2}$ and $\gamma = 2$.

We have generated initial isotropic straight-sided tetrahedral meshes with the same resolution, 1577 nodes. The meshes are of polynomial degree 1,2, and 4 and composed by 7296, 912 and 114 elements respectively. Perspectives of the mesh are shown in Figures 1(a), 1(b) and 1(c). They are colored according to the point-wise quality measure. The optimized meshes are presented in Figures 1(d), 1(e) and 1(f). We observe that the elements away from the anisotropic region are enlarged whereas the elements lying in the anisotropic region are compressed. In the optimized mesh the minimum is improved and the standard deviation of the element qualities is reduced when compared with the initial configuration.

To optimize the objective function presented in [2] we have applied, for each globalization strategy (BLS and RTR), the standard and proposed inexact Newton optimization methods. The optimization results are shown in Table I. We observe that, for the proposed optimization methods, the total amount of matrix vector products and the number of non-linear iterations have been reduced. Furthermore, we observe that the optimization results for the BLS and RTR strategy are comparable.

TABLE I. NON-LINEAR ITERATIONS, GLOBALIZATION ITERATIONS AND MATRIX-VECTOR PRODUCTS FOR STANDARD AND PROPOSED BACKTRACKING LINE-SEARCH (BLS) AND RESTRICTED TRUST-REGION (RTR) STRATEGIES.

Globalization strategy	Mesh degree	Non-linear iterations		Globalization iterations		Matrix-vector products	
		Standard	Proposed	Standard	Proposed	Standard	Proposed
BLS	1	52	32	83	50	5314	231
BLS	2	75	54	349	136	14043	572
BLS	4	91	84	445	163	16072	733
BLS	8	277	141	1211	319	18773	2424
RTR	1	251	35	0	72	22162	277
RTR	2	272	52	0	122	13061	471
RTR	4	305	80	0	157	15623	742
RTR	8	397	110	0	233	29872	1758

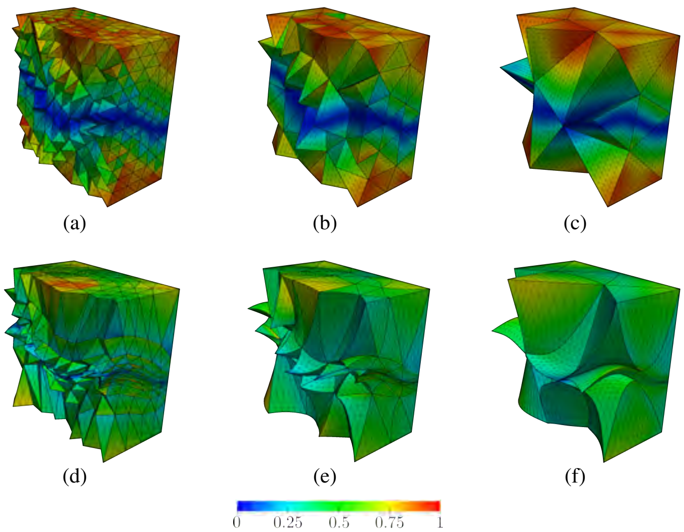

Fig. 1. Slices of tetrahedral meshes of polynomial degree 1, 2, and 4 in columns. Initial straight-sided isotropic meshes and optimized meshes from initial meshes in rows. Meshes are colored according to the pointwise quality measure presented in [2].

III. CONCLUSIONS

For r-adaption problems, standard implementations of BLS and RTR are not comparable while our new specific-purpose implementations are. To propose these two new implementations of BLS and RTR, we propose different emulation methods to interchange between both approaches their non-shared globalization features. Once the number of non-linear iterations is comparable, we have been able to improve the inexact Newton implementation, with both globalization methods, to reduce one order of magnitude the total number of sparse matrix-vector products.

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